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PROSPECTS FOR SUPERGRAVITY[†]

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Present particle phenomenology appears to be successfully described by a renormalizable gauge theory based on the low energy group

$$SU(3)_{\text{colour}} \times SU(2)_{\text{left}} \times U(1) \quad (1)$$

The three effective coupling constants of this theory vary with energy according to the renormalization group equations and become approximately equal at an energy of about 10^{14} - 10^{15} GeV, where one can attempt a grand unified theory (GUT) based on a gauge group with only one coupling constant and containing the low energy group(1). As discussed by Georgi and Glashow, the minimal GUT is that based on the group $SU(5)$. All other viable GUTs with larger groups proposed till now go through $SU(5)$. The GUT picture has had some successes, notably the determination, in reasonable agreement with experiment, of the low energy weak angle^{2,3,4} which parametrizes nuclear current couplings and certain relations^{3,4,5} between quark and lepton masses.

The grand unification energy is very large and not much smaller than the Planck mass $m_p \approx 10^{19}$ GeV, where gravity becomes important. A natural way to unify gravity with lower spin fields is provided by supergravity, the supersymmetric extension of Einstein's gravity. In working out this appealing idea one is faced with the difficulty that the largest supergravity theory (largest theory with a supermultiplet with maximum spin not exceeding two) is the $N=8$ theory (N is the number of supersymmetries) which has an $SO(8)$ internal symmetry. Now, the group $SO(8)$ is too small, it does not contain the low energy group (1), and the fundamental supermultiplet of the $N=8$ theory (which consists of 1 spin 2, 8 spin 3/2, 28 vectors, 56 spin 1/2 and 70 scalars) is not rich enough to accomodate all known quarks, leptons and spin-one gauge fields.

Perhaps these difficulties are only apparent and what is really important is that $SO(8)$ can contain $SU(3)_{\text{colour}} \times U(1)_{\text{e.m.}}$, which in our present picture is the exact gauge group of nature. It is a general feature of supergravity theories that the vector fields are exactly the right number and in the right representation to become the gauge fields of $SO(N)$. For $N \leq 4$ it has been possible to introduce a gauge coupling g for the vector fields. Supersymmetry requires it to be accompanied by a mass term $\propto g/k$ for the spin 3/2 fields and by a cosmological term $\propto (g/k^2)^2$. Perhaps this gauging can be achieved also for the $N=8$ theory. As suggested by Hawking, such an

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enormous cosmological constant could give rise to a foamy structure of space time. Over distances large compared to the Planck length the foamy structure would not be visible and space time could look smooth and approximately Minkowskian. At the same time the $SO(8)$ internal symmetry would be masked and a larger approximate effective symmetry could result. However interesting this point of view, it is hard to see how one would proceed to implement it. In the following we shall assume that the cosmological constant vanishes, due to cancellation among different contributions to it.

An approach which seems more promising at this time is that based on the observation that $N=8$ supergravity (without the gauge coupling g) has not only an $SO(8)$ symmetry but also a less obvious $SU(8)$ internal symmetry. This symmetry was observed first¹⁰ as a true global symmetry of the equations of motion. The complex transformations of the $SU(8)$ group are realized on the spin-one fields as duality rotations which transform the electric into the magnetic field strengths. More recently, Cremmer and Julia¹¹ have formulated the $N=8$ theory in such a way that the $SU(8)$ in a certain sense becomes local. They achieved this by adding redundant scalar fields (which could actually be eliminated by going to a particular gauge) and by introducing $SU(8)$ gauge vectors as auxiliary non-propagating fields, without a kinetic term of their own. Nevertheless, Cremmer and Julia suggested that the $SU(8)$ gauge vectors could become dynamical, i.e. their propagator develop a zero mass pole, in analogy with a similar phenomenon known to occur in the CP^{n-1} non-linear model in two space-time dimensions when treated in the large n approximation.¹² In effect one is assuming^{13,14} that the fields of the fundamental supermultiplet of $N=8$ supergravity bind together to form another supermultiplet containing spin-one fields in the adjoint representation of $SU(8)$. Just as the group $SU(8)$ is large enough to contain the group of grand unification, the supermultiplet of bound states can be sufficiently rich to contain all known quarks, leptons, vectors and scalars. All fields considered elementary at the present time are taken instead to be composite.

One then could have the following picture. At an energy comparable with (or higher than) the Planck mass m_P , supersymmetry is valid and also the larger internal symmetry $SU(8)$. The dynamics is specified by the gravitational constant $\kappa \sim 1/m_P$. Coming down to the GU mass, all supersymmetries are broken and $SU(8)$ is broken down to the GU group, say $SU(5)$. This symmetry breaking is spontaneous and characterized by the vacuum expectation value $\langle\phi\rangle$ of some scalar field. At the GU mass the interactions are determined by a dimensionless coupling constant $g \sim \kappa\langle\phi\rangle$ which emerges as the GU gauge coupling constant. The GUT appears as a low energy effective theory derived from the dynamics which is valid at the Planck mass. That it is a renormalizable gauge theory can be perhaps understood from the so called "zero mass decoupling theorem."

This theorem states that if a large mass (say m_P for definiteness) occurs in a field theory, and if a subset of fields or states remains massless as $m_P \rightarrow \infty$, then these states decouple from the rest in the limit and their effective interactions at low energy are renormalizable (with additional non-renormalizable interactions

inversely proportional to some power of m_p). The zero mass decoupling theorem is physically intuitive and can actually be proven, if the original field theory is a well defined renormalizable theory (for instance a gauge theory) and the zero mass states are those corresponding to some of the fields of the original theory.¹⁵ Presumably, it can also be proven for zero-mass bound states. Indeed, if their effective interactions were not renormalizable, divergences would arise in the computation of vertex functions, for which the only cut-off can be the inverse size of the bound states, which we expect to be m_p . This would result in masses of order m_p , or a breakdown of perturbation theory for the effective interactions of the bound states.* We wish to apply the theorem to the case where the original theory is $N=8$ supergravity and the zero mass states are composites of the fundamental fields. A consequence of the zero mass decoupling theorem is that the zero mass states cannot have spin higher than one, since their interaction would be necessarily non-renormalizable. For the same reason, the spin-one fields must be gauge fields and the zero mass spin one-half fields must form an anomaly free set. Scalar fields can survive in any number. All fields or states which do not belong to this subset with renormalizable interactions must disappear as $m_p \rightarrow \infty$, by acquiring very large masses (or perhaps by becoming unbound).

According to Cremmer and Julia, the composite fields of the $SU(8)$ gauge vectors are expressions (bilinear plus higher order terms) in the fundamental scalar fields. These expressions are not identical with the currents of the global $SU(8)$ transformations mentioned above, which differ from them by the presence of terms containing the other fundamental fields (spin $1/2$, 1 and $3/2$) and which furthermore are non-local for the spin 1 , on which they generate duality rotations. Nevertheless, the gauge $SU(8)$ and the global $SU(8)$ are clearly related and could be identified by going to a particular fixed gauge, where only the global $SU(8)$ remains. Ellis, Gaillard, Maiani and I^{13,14} have reinterpreted the suggestion of Cremmer and Julia and have tried to determine the supermultiplet containing the conserved currents of the global $SU(8)$. For a massless supersymmetric system with $N \leq 4$ and maximum spin 1 , all currents are local and this supermultiplet would be the supercurrent supermultiplet,¹⁶ which contains also the energy momentum tensor and the N spinor currents. One can put the fields of this supermultiplet on the mass shell and one obtains in this way a massive supermultiplet of supersymmetry with $N \leq 4$ spinor charges. The maximum spin is 2 and is a singlet. States with lower spins are in representations of $Sp(2N)$. As the mass tends to zero the supermultiplet breaks up into a finite number of massless supermultiplets in which the various helicities are in representations of $SU(N)$. It is not difficult to identify which of these massless supermultiplets contains the $SU(8)$ currents in the adjoint representation and one finds that it is of the type given below in (2), (3). This argument is reliable for $N \leq 4$; we assume that the multiplet has the same form for $N=8$,

*This argument is due to M. Veltman.

We come thus to the conclusion that the relevant supermultiplet of states is given by $(A, B, \dots = 1, 2, \dots, 8)$

$$\left(\frac{3}{2}\right)_A^A, (1)_B^A, \left(\frac{1}{2}\right)_{[BC]}^A, (0)_{[BCD]}^A, \dots, \left(-\frac{5}{2}\right)_A^A, \quad (2)$$

to which one must add the TCP conjugate states

$$\left(\frac{5}{2}\right)_A, (2)_{AB}, \left(\frac{3}{2}\right)_{A[BC]}, \dots, \left(-\frac{3}{2}\right)_A, \quad (3)$$

(a set of k antisymmetrized lower indices is equivalent to $8-k$ antisymmetrized upper indices). It contains spin-one states in the adjoint representation but also other spin-one states. Ideally one would like to give masses by super-Higgs effect to all unwanted states, those which do not belong to the surviving zero mass subset with effective renormalizable interactions. However, it is easy to see that this cannot be done in an $SU(5)$ invariant way, and not even in an $SU(3) \times SU(2) \times U(1)$ invariant way. For instance, in order to give an invariant mass to a spin $5/2$ state, one needs all the helicities $5/2, 3/2, 1/2, -1/2, -3/2, -5/2$ and they must be in the same representation of the invariance group. It is clear that the supermultiplet (2), (3) does not contain all the necessary helicities. It is also easy to convince oneself that no irreducible supermultiplet and no finite set of irreducible supermultiplets will do.* Clearly, this is due to our desire to be left with complex (chiral) massless representations; it is very easy to construct finite sets of massless supermultiplets which combine to massive ones, in a vector-like way, simply by starting from a finite mass supermultiplet and going to the limit as the mass tends to zero when it breaks up into massless ones.

The above discussion suggests that one may need an infinite dimensional set of irreducible supermultiplets. This would mean that the dynamics at the Planck mass involves states of arbitrarily high spin most of which become infinitely massive after the symmetry breaking, as $m_p \propto \langle \phi \rangle \rightarrow \infty$. Those higher spin states which, like the graviton, remains massless, will be left with non-renormalizable interactions proportional to reciprocal powers of m_p .

*A formal proof of this impossibility has been worked out by M. Gell-Mann, private communication. In a recent Harvard preprint, HUTP-80/A050, P. Frampton considers finite sets of supermultiplets and states that his solutions have enough helicity states to give masses to all higher spin states. However these masses are not $SU(3)_{\text{colour}}$ invariant and would give rise to breaking of this group already at energies comparable with the Planck mass.

In an attempt to extract as many properties of the GUT as possible from the dynamics at the Planck mass, without actually knowing this dynamics (nor the possibly infinite-dimensional multiplet), Ellis, Gaillard and I¹⁴ have assumed that all surviving low energy states are already contained in the single supermultiplet (2), (3). We have then looked for the maximal set of states of spin 0, 1/2 and 1 contained in it which can have renormalizable interactions, without however keeping those SU(8) representations which are obtained by saturating in (2), (3) upper with lower indices (traces). With these (admittedly very drastic) simplifications, we found that the breaking of SU(8) to SU(5) (rather than SU(6) or SU(7)) is preferred. For the maximal set of left-handed states we found two solutions, of which only one is vector-like for SU(3)_{colour} × SU(1)_{e.m.}. This solution has three generations of 10 + $\bar{5}$ of SU(5).

Irrespective of the validity of this particular set of states, one may ask by what techniques one can obtain more information about the GUT. Now, in our picture, dynamics at the Planck mass involves higher spin states. It appears at this moment, that no consistent classical local Lagrangian describing the interaction of higher spin fields exists. Therefore the only available technique for describing these interactions is that of on-shell scattering amplitudes (S-matrix). It is not difficult¹⁷ to find the constraints imposed on these amplitudes by supersymmetry and it seems also possible to formulate the spontaneous breaking process. In this way one can obtain, in principle, not only the group representations of the GUT but also the details of the interaction, the Higgs potential and the Yukawa coupling. Perhaps this approach can give some understanding of the so-called hierarchy problem.¹⁸

Incidentally, having taken the point of view that higher spins are entering anyway the dynamics at (or above) the Planck mass one may even try to relax the restriction to supersymmetries with $N < 8$ and admit spins higher than 2 in the fundamental supermultiplet. Our present point of view that local Lagrangians are low energy approximations to a theory formulated in terms of on shell amplitudes seems to give us this freedom. Still, there seems to be some virtue in keeping with $N=8$ supergravity, a theory remarkable for its convergence and its symmetries.

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